International Journal of Innovative Research in Technology
(An International Open Access , Peer-reviewed, Refereed Journal)
IJIRTEXPLORE- Search Thousands of research papers
Call For Paper May 2022 Last Date 25 - May 2022
Impact Factor 7.376 (Year 2021)
ISSN: 2349-6002
ESTD Year: 2014
UGC approved journal no 47859

© February 2018 | IJIRT | Volume 4 Issue 9 | ISSN: 2349-6002

I–Convergence of Ultra filters

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***Abstract*- In this paper, we have extended the idea of I–convergence of filters to the I–convergence of ultra-filters containing that filter and studied its various properties.**

I. INTRODUCTION

The concept of convergence of a sequence of real numbers has been extended to statistical convergence independently by H. Fast [4] and I. J. Schoenberg [24]. Kostyrko et. al in [10] and [11] generalized the notion of statistical convergence and introduced the concept of I–convergence of real sequences which is based on the structure of the ideal I of subsets of the set of natural numbers. Mursaleen et. al [16] defined and studied the notion of ideal convergence in random 2–normed spaces and construct some interesting examples. Several works on I–convergence and statistical convergence have been done in [1], [3], [6], [7], [8], [9], [10], [11], [12], [15], [16], [17], [18], [19], [23].

The idea of I–convergence has been extended from real number space to metric space [10] and to a normed linear space [22] in recent works. Later the idea of I–convergence was extended to an arbitrary topological space by B. K. Lahiri and P. Das in [13]. It was observed that the basic properties remained preserved in topological spaces. Lahiri and Das [14] introduced the idea of I–convergence of nets in topological spaces and examined how far it affects the basic properties.

Taking the idea of [14], Jamwal et. al introduced the idea of I–convergence of filters in [6] and studied its various properties. Jamwal et. al reintroduced the idea of I–convergence of nets in topological spaces and established the equivalence of I–convergences of nets and filters on topological spaces in [7]. In [8], Jamwal et. al introduced the idea of I–cluster point of filters and studied its various properties. Jamwal et. al established the equivalence of I–cluster points of filters and cluster points of nets as well as the equivalence of I–cluster points of filters and nets in [9].

We start with the following definitions:

Definition 1.1 Let X be a non-empty set. Then a family $\mathfrak{F} \subset 2^X$ is called a *filter* on X if
(i) $\phi \notin \mathfrak{F}$,
(ii) $A, B \in \mathfrak{F}$ implies $A \cap B \in \mathfrak{F}$ and
(iii) $A \in \mathfrak{F}, B \supset A$ implies $B \in \mathfrak{F}$.

Definition 1.2 Let X be a non-empty set. Then a family $I \subset 2^X$ is called an *ideal* of X if
(i) $\phi \in I$,
(ii) $A, B \in I$ implies $A \cup B \in I$ and
(iii) $A \in I, B \subset A$ implies $B \in I$.

Definition 1.3 Let X be a non-empty set. Then a filter $\mathfrak{F}$ on X is said to be *non-trivial* if $\mathfrak{F} \neq \{X\}$.

Definition 1.4 Let X be a non-empty set. Then an ideal I of X is said to be *non-trivial* if $I \neq \{\phi\}$ and $X \notin I$.

Note (i) $\mathfrak{F} = \mathfrak{F}(I) = \{A \subset X : X \setminus A \in I\}$ is a filter on X, called the *filter associated with the ideal* I.
(ii) $I = I(\mathfrak{F}) = \{A \subset X : X \setminus A \in \mathfrak{F}\}$ is an ideal of X, called the *ideal associated with the filter* $\mathfrak{F}$.
(iii) A non-trivial ideal I of X is called *admissible* if I contains all the singleton subsets of X.
Several examples of non-trivial admissible ideals have been considered in [10].
Throughout this paper, X will stand for a topological space and $I = I(\mathfrak{F})$ will be the ideal associated with the filter $\mathfrak{F}$ on X.
We give a brief discussion on I–convergence and I–cluster points of filters and nets in topological spaces as given by [6], [7], [8], [9].

Definition 1.5 A filter $\mathfrak{F}$ on X is said to be *I–convergent* to $x_0 \in X$ if for each nbd U of x_0, $\{y \in X : y \notin U\} \in I$.
In this case, x_0 is called an *I–limit* of $\mathfrak{F}$ and is written as $I - \lim \mathfrak{F} = x_0$.

Definition 1.6 A point $x_0 \in X$ is called an *I–cluster point* of a filter $\mathfrak{F}$ on X if for each nbd U of x_0, $\{y \in$