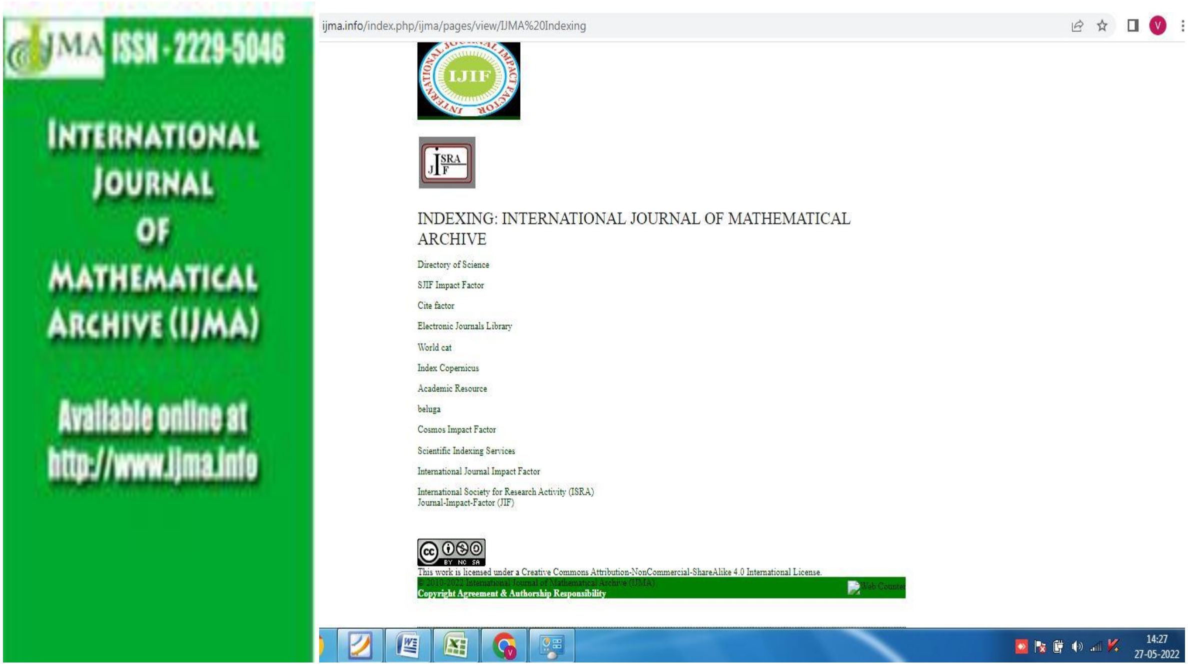

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ON DYNAMICAL SYSTEM AND SEMIGROUPS INDUCED BY COMPOSITE CONVOLUTION OPERATORS

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ABSTRACT

In this paper, we analyse Composite Convolution operators which are obtained by composing convolution operators with composition operators. We calculate the norm of composite convolution operators. The norm of trace of composite convolution operators has also been explored. In this paper, an attempt has been made to investigate semigroups of one-parameter family and two-parameter family of composite convolution operators. A dynamical system induced by composite convolution operator is also obtained.

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Keywords: *Composition operator, Composite Convolution operator, Radon- Nikodym derivative, Expectation operator Semigroup, Dynamical system.*

1. INTRODUCTION

Let (X, Ω, μ) be a σ-finite measure space. For $1 \leq p < \infty$ and for each $f \in L^p(\mu)$, there exists a unique $\phi^{-1}(\Omega)$ measurable function E(f) such that

$$\int gf\, d\mu = \int g\, E(f) d\mu$$

for every $\phi^{-1}(\Omega)$ measurable function g for which left integral exists. The function E(f) is called conditional expectation of f with respect to the sub - algebra $\phi^{-1}(\Omega)$. For more details about expectation operators, one can refer to Parthasarthy [8]. Let $\phi: X \to X$ be a non-singular measurable transformation, (i.e., $\mu(E) = 0 \Rightarrow \mu\phi^{-1}(E) = 0$). Then a composition transformation, for $1 \leq p < \infty$, $C_\phi: L^p(\mu) \to L^p(\mu)$ is defined by

$$C_\phi = f\, o\phi$$

for every $f \in L^p(\mu)$. If C_ϕ is continuous, then it is a composition operator induced by ϕ. It is well-known result of Singh [11] that C_ϕ is a bounded operator if and only if $f_0 = \frac{d\mu\phi^{-1}}{d\mu}$, the Radon-Nikodym derivative of the measure $\mu\phi^{-1}$ with respect to the measure μ is essentially bounded. For more detail about composition operators, we refer to Singh and Manhas [11].

Given $f, g \in L^2(\mathbb{R})$, then convolution of f and g, $f * g$ is defined by

$$f * g(x) = \int g(x - y) f(y)\, d(y).$$