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FATOU AND JULIA LIKE SETS

KULDEEP SINGH CHARAK, ANIL SINGH, AND MANISH KUMAR

ABSTRACT. For a family of holomorphic functions on an arbitrary domain, we introduce Fatou and Julia like sets, and establish some of their interesting properties.

1. INTRODUCTION AND MAIN RESULTS

Throughout, we shall denote by $\mathcal{H}(D)$ the class of all holomorphic functions on a domain $D \subseteq \mathbb{C}$. A subfamily $\mathcal{F}$ of $\mathcal{H}(D)$ is said to be normal if every sequence in $\mathcal{F}$ contains a subsequence that converges locally uniformly on D. $\mathcal{F}$ is said to be normal at a point $z_0 \in D$ if it is normal in some neighborhood of z_0 in D (see [12, 15]).

Let f be an entire function and let $f^n := \underbrace{f \circ f \circ \cdots \circ f}_{n-times}(n \geq 1)$ be the n-th iterate of f. The *Fatou set* of f, denoted by $F(f)$, is defined as

$$F(f) = \{z \in \mathbb{C} : \{f^n\} \text{ is a normal family in some neighborhood of } z\}$$

and, the complement $\mathbb{C} \setminus F(f)$ of $F(f)$ is called the *Julia set* of f and is denoted by $J(f)$. $F(f)$ is an open subset of $\mathbb{C}$ and $J(f)$ is a closed subset of $\mathbb{C}$, and both are completely invariant sets under f. The study of Fatou and Julia sets of holomorphic functions is a subject matter of Complex Dynamics for which one can refer to [2, 4, 14].

For a given domain D and a subfamily $\mathcal{F}$ of $\mathcal{H}(D)$, we denote by $F(\mathcal{F})$, a subset of D on which $\mathcal{F}$ is normal and $J(\mathcal{F}) := D \setminus F(\mathcal{F})$. If $\mathcal{F}$ happens to be a family of iterates of an entire function f, then $F(\mathcal{F})$ and $J(\mathcal{F})$ reduce to the Fatou set of f and the Julia set of f respectively, therefore, it is reasonable to call $F(\mathcal{F})$ and $J(\mathcal{F})$ as *Fatou and Julia like sets*. Note that Julia set of an entire function is always non-empty (see [2]) whereas Julia like set $J(\mathcal{F})$ can be empty. For example, consider the family

$$\mathcal{F} := \{f(az+b) : a, b \in \mathbb{C}, a \neq 0\},$$

where f is a normal function on $\mathbb{C}$ (see [12], p. 179). Then since f is a normal function on $\mathbb{C}$, $\mathcal{F}$ is a normal family on $\mathbb{C}$. That is, $F(\mathcal{F}) = \mathbb{C}$ and hence $J(\mathcal{F}) = \phi$.

Also, it is interesting to note that Julia set of any meromorphic function is an uncountable set (see [2]) but Julia like set is not so, for example, $J(\mathcal{F}) = \{0\}$, where $\mathcal{F} := \{nz : n \in \mathbb{N}\} \subset \mathcal{H}(\mathbb{D})$, where $\mathbb{D}$ is the open unit disk.

If $\mathcal{F}$ and $\mathcal{G}$ are two subfamilies of $\mathcal{H}(D)$, then $J(\mathcal{F} \cap \mathcal{G}) \subset J(\mathcal{F}) \cap J(\mathcal{G})$, however $J(\mathcal{F} \cap \mathcal{G}) = J(\mathcal{F}) \cap J(\mathcal{G})$ may not hold in general. For example, let

$$\mathcal{F} = \{nz : n \in \mathbb{N}\} \cup \{n(z-1) : n \in \mathbb{N}\}$$

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