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α-B-Finitisticness of Fuzzy Bitopological Spaces

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Abstract:
In this paper, we have introduced the concept of α-B-finitistic fuzzy bitopological spaces and studied some of their basic properties.

Keywords:
Covering Dimension, Finitisticness, Fuzzy Bitopological Space, Open Refinement.

1. Introduction and preliminaries

The order of a family $\{U_\lambda : \lambda \in \Delta\}$ of subsets, not all empty, of some set X is the largest integer n for which there exists a subsets M of Δ with $n+1$ elements such that $\bigcap_{\lambda \in M} U_\lambda$ is non-empty, or is ∞ if there is no such largest integer.

Let $\Delta \neq \emptyset$ and $\mathcal{A} = \{A_\lambda : \lambda \in \Delta\}$ be a family of fuzzy subsets of a non-empty set X. Then order of $\mathcal{A}$ is defined as under:

Case I. When $A_\lambda \neq \underline{0}$ for atleast one value of λ in Δ. Then the order of $\mathcal{A}$ is the largest non-negative integer n for which there exists a subset M of Δ having $n+1$ elements such that $\bigwedge_{\lambda \in M} A_\lambda \neq \underline{0}$ or is ∞ if there is no such largest integer n.

Case II. When $A_\lambda = \underline{0}$ for all $\lambda \in \Delta$. Then the order of $\mathcal{A}$ is -1.

The concept of bitopological space was introduced by Kelly [6]. A bitopological space is a triplet (X, τ_1, τ_2) where X is a non-empty set and τ_1, τ_2 are two topologies on X. Let (X, τ_1, τ_2) be a bitopological space. A subfamily $\{U_\lambda : \lambda \in \Lambda\}$ of τ_i is said to be τ_i open cover of (X, τ_1, τ_2) where $i = 1, 2$ if $\bigcup_{\lambda \in \Lambda} U_\lambda = X$. A bitopological space

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(X,τ_1,τ_2) is said to be B-compact if each τ_i open cover of X has τ_j finite subcover where $i,j=1,2$ and $i\neq j$. Let (X,τ_1,τ_2) and (Y,τ_3,τ_4) be two bitopological spaces. A function $f:(X,\tau_1,\tau_2)\to(Y,\tau_3,\tau_4)$ is said to be B-continuous if inverse image of every τ_3 open subset of Y is τ_2 open subset of X and inverse image of every τ_4 open subset of Y is τ_1 open subset of X. A function $f:(X,\tau_1,\tau_2)\to(T,\tau_3,\tau_4)$ is said to be B-homeomorphism if both f and f^{-1} are B-continuous. A general bitopological space (X,τ_1,τ_2) is said to be B-finitistic if each τ_i open cover of X has τ_j finite order open refinement where $i,j=1,2$ and $i\neq j$.

Any function $A:X\to I$ where $I=[0,1]$ is called a fuzzy subset of X. The set of all fuzzy subsets of X is denoted by I^X. A subfamily $\delta\subset I^X$ is said to be a fuzzy topology on X if

(i) $\underline{0},\underline{1}\in\delta$,

(ii) $\{U_\lambda:\lambda\in\Lambda\}\subset\delta\Rightarrow\bigvee_{\lambda\in\Delta}U_\lambda\in\delta$,

(iii) $U,V\in\delta\Rightarrow U\wedge V\in\delta$.

The pair (X,δ) is called fuzzy topological space. For every $a\in I$, $\underline{a}$ is called "a" valued constant function from X to I. A fuzzy subset A is called a crisp subset if there exists an ordinary subset U of X such that $A=\chi_U$, where $\chi_U:X\to\{0,1\}\subset I$ is the characteristic function of U. The family of all the crisp subsets contained in δ is denoted by $crs(\delta)$ and $[\delta]$ is defined as $[\delta]=\{U\subset X:\chi_U\in crs(\delta)\}$. For a fuzzy topological space (X,δ), $crs(\delta)$ is a fuzzy topology on X and $[\delta]$ is general topology on X. A fuzzy bitopological space is a triplet (X,τ_1,τ_2), where X is a non-empty set and δ_1, δ_2 are two fuzzy topologies on X. Let (X,δ_1,δ_2) be a fuzzy bitopological space. A subfamily $\{U_\lambda:\lambda\in\Lambda\}$ of δ_i is said to be δ_i open cover of (X,δ_1,δ_2) where $i=1,2$ if $\bigvee_{\lambda\in\Lambda}U_\lambda=\underline{1}$. Let (X,δ) be a fuzzy topological space. For every $\alpha\in[0,1)$, a subfamily $\mathcal{U}$ of δ is said to be an α-open cover of (X,δ) if for every $x\in X$, there exists some $U\in\mathcal{U}$ such that $U(x)>\alpha$ (page no.187 of [7]). An α-open cover is also called α-shading.

2. α-B-finitistic fuzzy bitopological spaces

Definition 2.1. A fuzzy bitopological space (X,δ_1,δ_2) is said to be α-B-finitistic if each δ_i α-open cover of (X,δ_1,δ_2) has δ_j finite order α-open refinement where $i\neq j$ and $i,j=1,2$.

Theorem 2.2. *A general bitopological space* (X,τ_1,τ_2) *is* B *-finitistic if and only if* $(X,\chi(\tau_1),\chi(\tau_2))$ *is* α-B *-finitistic.*

Proof. Here (X,τ_1,τ_2) is a general bitopological space and $(X,\chi(\tau_1),\chi(\tau_2))$ is a fuzzy bitopological space where $\chi(\tau_1)=\{\chi_V : V\in\tau_1\}$ and $\chi(\tau_2)=\{\chi_V : V\in\tau_2\}$. Suppose (X,τ_1,τ_2) is B -finitistic. We have to show that $(X,\chi(\tau_1),\chi(\tau_2))$ is α-B -finitistic. Let $\mathcal{U}=\{\chi_{U_\lambda} : \lambda\in\Delta\}$ be any $\chi(\tau_i)$ α -open cover of $(X,\chi(\tau_1),\chi(\tau_2))$. We claim that $\mathcal{V}=\{U_\lambda : \chi_{U_\lambda}\in\mathcal{U}\}$ is a τ_i open cover of (X,τ_1,τ_2). Let $x\in X$. Since $\mathcal{U}$ is $\chi(\tau_i)\alpha$ -open cover of $(X,\chi(\tau_1),\chi(\tau_2))$, there exists some $\chi_{U_\lambda}\in\mathcal{U}$ such that $\chi_{U_\lambda}(x)>\alpha$. But $\chi_{U_\lambda}(x)>\alpha\Rightarrow\chi_{U_\lambda}(x)>0\Rightarrow\chi_{U_\lambda}(x)=1\Rightarrow x\in U_\lambda$. This means that $X=\bigcup_{\lambda\in\Delta}U_\lambda$. This shows that $\mathcal{V}$ is τ_i open cover of (X,τ_1,τ_2). Since (X,τ_1,τ_2) is B -finitistic, therefore $\mathcal{V}$ has a τ_j finite order open refinement, say $\mathcal{W}=\{W_t : t\in\Delta_1\}$. We shall show that $\mathcal{S}=\{\chi_{W_t} : W_t\in\mathcal{W}\}$ is a $\chi(\tau_j)$ finite order α -open refinement of $\mathcal{U}$. We first show that $\mathcal{S}$ is $\chi(\tau_j)\alpha$ -open cover. Since $\mathcal{W}$ is τ_j open cover of X, $\bigcup_{t\in\Delta_1}W_t=X$. Now, $\bigcup_{t\in\Delta_1}W_t=X\Rightarrow\chi_{\bigcup_{t\in\Delta_1}W_X}=\chi_X=\underline{1}\Rightarrow\bigvee_{t\in\Delta_1}\chi_{W_t}=\underline{1}\Rightarrow$ $\bigvee_{t\in\Delta_1}\chi_{W_t}(x)=\underline{1}(x)=1>\alpha$, $\forall x\in X$. This means that for all $x\in X$, there exists $\chi_{W_t}\in\chi(\tau_j)$ such that $\chi_{W_t}(x)>\alpha$. Therefore, $\mathcal{S}$ is $\chi(\tau_j)\alpha$ -open cover of X. Since $\mathcal{W}$ is τ_j refinement of $\mathcal{V}$, for every $W_t\in\mathcal{W}$, there exists $U_\lambda\in\mathcal{V}$ such that $W_t\in U_\lambda$. Clearly, $\chi_{W_t}\le\chi_{U_\lambda}$. Thus for each $\chi_{W_t}\in\mathcal{S}$, there exists $\chi_{U_\lambda}\in\mathcal{U}$ such that $\chi_{W_t}\le\chi_{U_\lambda}$. Hence $\mathcal{S}$ is $\chi(\tau_j)\alpha$ -open refinement of $\mathcal{U}$. Finally, suppose order of $\mathcal{W}=n$ ($\because\mathcal{U}$ is of finite order). Then $\bigcap_{i=1}^{n+2}W_{t_i}=\emptyset\Rightarrow\chi_{\bigcap_{i=1}^{n+2}W_{t_i}}=\underline{0}\Rightarrow\bigwedge_{i=1}^{n+2}\chi_{W_{t_i}}=\underline{0}$. This shows that $\mathcal{S}$ is also of finite order. Thus $\mathcal{S}$ is $\chi(\tau_j)$ finite order α -open refinement of $\mathcal{U}$. Hence $(X,\chi(\tau_1),\chi(\tau_2))$ is α-B -finitistic.

Conversely, let $(X,\chi(\tau_1),\chi(\tau_2))$ be α-B -finitistic. We have to show that (X,τ_1,τ_2) is B -finitistic. Let $\mathcal{U}=\{U_\lambda : \lambda\in\Delta\}$ be any τ_i open cover of (X,τ_1,τ_2). We show that $\mathcal{V}=\{\chi_{U_\lambda} : U_\lambda\in\mathcal{U}\}$ is $\chi(\tau_i)\alpha$ -open cover of $(X,\chi(\tau_1),\chi(\tau_2))$. For this, let $x\in X$. Since $\mathcal{U}=\{U_\lambda : \lambda\in\Delta\}$ is any τ_i open cover of (X,τ_1,τ_2), there exists some $U_\beta\in\mathcal{U}$ such that $x\in U_\beta$. Then $\chi_{U_\beta}(x)=1>\alpha$. Hence $\mathcal{V}=\{\chi_{U_\lambda} : U_\lambda\in\mathcal{U}\}$ is $\chi(\tau_i)\alpha$ -open cover of $(X,\chi(\tau_1),\chi(\tau_2))$. Since $(X,\chi(\tau_1),\chi(\tau_2))$ is α-B -finitistic, therefore $\mathcal{V}$ has $\chi(\tau_j)$ finite order α -open refinement, say $\mathcal{W}=\{\chi_{W_t} : t\in\Delta\}$. Then clearly $\mathcal{S}=\{W_t : \chi_{W_t}\in\mathcal{W}\}$ is τ_j finite order open refinement of $\mathcal{U}$. Hence (X,τ_1,τ_2) is B -finitisitc.

Theorem 2.3. *Let (X,δ_1,δ_2) be a fuzzy bitopological space. Then $(X,[\delta_1],[\delta_2])$ is B-finitistic if and only if $(X,crs\delta_1,crs\delta_2)$ is α-B-finitistic.*

Proof. We know that $crs(\delta_i)$ $(i=1,2)$ is a fuzzy topology on X and $[\delta_i]$ is a general topology on X. Thus the result follows by Theorem 2.2.

Theorem 2.4. *A general bitopological space (X,τ_1,τ_2) is B-finitistic if and only if (X,δ_1,δ_2) is α-B-finitistic, where $\delta_1=(\underline{U}:U\in\tau_1)$ and $\delta_2=(\underline{U}:U\in\tau_2)$. Here $\underline{U}$ denotes the constant U function from X to I.*

Proof. Suppose (X,τ_1,τ_2) is B-finitistic. We shall show that (X,δ_1,δ_2) is α-B-finitisitc. For this, let $\mathcal{U}=\{\underline{U_\lambda}:\lambda\in\Delta\}$ be a $\delta_i\alpha$-open cover of (X,δ_1,δ_2). We first show that $\mathcal{V}=\{U_\lambda:\underline{U_\lambda}\in\mathcal{U}\}$ is τ_i open cover of (X,τ_1,τ_2).

Let $x\in X$. Since $\mathcal{U}$ is a $\delta_i\alpha$-open cover of X, there exists some $\underline{U_\lambda}\in\mathcal{U}$ such that $\underline{U_\lambda}(x)>\alpha$. Now, $\underline{U_\lambda}(x)>\alpha\Rightarrow\bigvee_{\lambda\in\Delta}\underline{U_\lambda}=1=\underline{1}(x)\Rightarrow\bigcup_{\lambda\in\Delta}U_\lambda=X$.

This shows that $\mathcal{V}$ is τ_i open cover of (X,τ_1,τ_2). Since (X,τ_1,τ_2) is B-finitistic, therefore $\mathcal{V}$ has τ_j finite order open refinement, say $\mathcal{W}=\{W_t:t\in\Lambda\}$. We shall show that $\mathcal{S}=\{\underline{W_t}:W_t\in\mathcal{W}\}$ is a δ_j finite order α-open refinement of $\mathcal{U}$.

Let $x\in X$. Then $\bigcup_{t\in\Lambda}W_t=X\Rightarrow\bigvee_{t\in\Lambda}\underline{W_t}(x)=\underline{1}(x)=1$. This means that there exists some $\underline{W_t}\in\mathcal{S}$ such that $W_t(x)>\alpha$. Also, let $\underline{W_t}\in\mathcal{S}$. Then $W_t\in\mathcal{W}$.

Since $\mathcal{W}$ is a τ_j open refinement of $\mathcal{V}$, there exists some $U_\lambda\in\mathcal{V}$ such that $W_t\subset U_\lambda$. But $W_t\subset U_\lambda$ implies $\underline{W_t}\le\underline{U_\lambda}$. This shows that $\mathcal{S}$ is $\delta_j\alpha$-open refinement of $\mathcal{U}$.

Finally, we show that $\mathcal{S}$ is of finite order.

Let order of $\mathcal{W}=n$. Let $\mathcal{S}_1$ be any subfamily of $\mathcal{S}$ having $n+2$ members. Then $\left(\bigwedge_{\underline{W_t}\in\mathcal{S}_1}W_t\right)(x)=\bigwedge_{\underline{W_t}\in\mathcal{S}_1}W_t(x)=\bigcap_{W_t\in\mathcal{S}_1}W_t=\emptyset=0=\underline{0}(x)$ implies $\bigwedge_{\underline{W_t}\in\mathcal{S}_1}W_t=\underline{0}$. This shows that order of $\mathcal{S}$ is not exceeding n. Hence (X,δ_1,δ_2) is α-B-finitistic.

Conversely, suppose (X,δ_1,δ_2) is α-B-finiitistic. We have to show that (X,τ_1,τ_2) is B-finitistic. Let $\mathcal{U}=\{U_\lambda:\lambda\in\Delta\}$ be τ_i open cover of (X,τ_1,τ_2). We shall show that $\mathcal{V}=\{\underline{U_\lambda}:U_\lambda\in\mathcal{U}\}$ is δ_i open cover of (X,δ_1,δ_2). Since $\mathcal{U}$ is τ_i open cover of X, $\bigcup_{\lambda\in\Delta}U_\lambda=X=\underline{1}$. Now, $\bigcup_{\lambda\in\Delta}U_\lambda=\underline{1}\Rightarrow\bigcup_{\lambda\in\Delta}U_\lambda(x)=\underline{1}(x)$, $\forall x\in X\Rightarrow\bigvee_{\lambda\in\Delta}\underline{U_\lambda}(x)=1>\alpha$, $\forall x\in X$. This means that there exits $\lambda\in\Delta$ such that $\underline{U_\lambda}(x)>\alpha$.

Thus $\mathcal{V}$ is $\delta_i\alpha$-open cover of (X,δ_1,δ_2). Since X is α-B-finitistic, $\mathcal{V}$ has a δ_j finite order α-open refinement, say $\mathcal{V}_1=\{\underline{V_t}:t\in\Lambda\}$. It can be easily checked that

$\mathcal{U}_1 = \{V_t : \underline{V_t} \in \mathcal{V}_1\}$ is a τ_j finite order open refinement of $\mathcal{U}$. Hence (X, τ_1, τ_2) is B-finitistic.

Theorem 2.5. *Let (X, δ_1, δ_2) be a α-B -finitistic fuzzy bitopological space and $(Y, \delta_1|_Y, \delta_2|_Y)$ be a B -closed subspace of (X, δ_1, δ_2). Then $(Y, \delta_1|_Y, \delta_2|_Y)$ is α-B -finitistic.*

Proof. Here (X, δ_1, δ_2) is a α-B -finitistic fuzzy bitopological space and $(Y, \delta_1|_Y, \delta_2|_Y)$ is B -closed subspace of (X, δ_1, δ_2). We have to show that $(Y, \delta_1|_Y, \delta_2|_Y)$ is α-B -finitistic. Let $\mathcal{U} = \{U_\lambda : \lambda \in \Lambda\}$ be any $\delta_i|_Y$ α -open cover of $(Y, \delta_1|_Y, \delta_2|_Y)$. Then each $U_\lambda = V_\lambda|_Y$, for some $V_\lambda \in \delta_i$ where $i = 1, 2$. We show that $\mathcal{V} = \{V_\lambda : U_\lambda = V_\lambda|_Y, \forall U_\lambda \in \mathcal{U}\} \cup \{\chi_{Y'}\}$ is $\delta_i\alpha$ -open cover of X. Let $x \in X$. Then $x \in Y$ or $x \in Y'$.

Case I. If $x \in Y$, then there exists some $U_\lambda \in \mathcal{U}$ such that $U_\lambda(x) > \alpha$. Then clearly $V_\lambda \in \mathcal{V}$ such that $V_\lambda(x) = U_\lambda(x) > \alpha$, where $U_\lambda = V_\lambda|_Y$ and $V_\lambda \in \{V_\lambda : U_\lambda = V_\lambda|_Y,$ $\forall U_\lambda \in \mathcal{U}\}$. Thus $V_\lambda(x) > \alpha$.

Case II. If $x \in Y'$, then $\chi_{Y'} \in \mathcal{V}$ such that $\chi_{Y'}(x) = 1 > \alpha$.

Hence $\mathcal{V} = \{V_\lambda : U_\lambda = V_\lambda|_Y, \forall U_\lambda \in \mathcal{U}\} \cup \{\chi_{Y'}\}$ is $\delta_i\alpha$ -open cover of (X, δ_1, δ_2) in both the cases.

Since (X, δ_1, δ_2) is α-B -finitistic, therefore $\mathcal{V}$ has δ_j finite order α -open refinement, say $\mathcal{V}_1 = \{W_\alpha : \alpha \in \Delta\}$. Then clearly $\mathcal{U}_1 = \{W_\alpha|_Y : W_\alpha \in \mathcal{V}_1\}$ is $\delta_j|_Y$ finite order α -open refinement of $\mathcal{U}$. Hence $(Y, \delta_1|_Y, \delta_2|_Y)$ is α-B -finitistic.

Theorem 2.6. *Let (X, τ_1, τ_2) be a general bitopological space and $Y \subset X$. Then $(Y, \tau_1|_Y, \tau_2|_Y)$ is B -finitistic if and only if $(Y, \chi(\tau_1)|_Y, \chi(\tau_2)|_Y)$ is α-B -finitistic.*

Proof. We know that $\chi(\tau_i)|_Y = \{\chi_U : U \in \tau_i|_Y\}$. Thus by Theorem 2.2 $(Y, \tau_1|_Y, \tau_2|_Y)$ is B -finitistic if and only if $(Y, \chi(\tau_1)|_Y, \chi(\tau_2)|_Y)$ is α-B -finitistic.

Remark 2.7. An arbitrary subspace of α-B -finitistic fuzzy bitopological space need not be α-B -finitistic.

We know that in general topology an arbitrary subspace of a finitistic space need not be finitistic [4]. Let (X, τ_1, τ_2) be a B -finitistic general bitopological space. Let $(Y, \tau_1|_Y, \tau_2|_Y)$ be a subspace of (X, τ_1, τ_2) which is not B -finitistic. Since (X, τ_1, τ_2) is B -finitistic, by Theorem 2.2 $(X, \chi(\tau_1)|_Y, \chi(\tau_2)|_Y)$ is α-B -finitistic. Also, by

Theorem 2.2, $\left(Y, \tau_1|_Y, \tau_2|_Y\right)$ is not B -finitistic implies $\left(Y, \chi(\tau_1)|_Y, \chi(\tau_2)|_Y\right)$ is not α-B -finitistic.

Example 2.8. Let X be a non-empty set and $a \in [0,1)$. Let $\delta_1 = \delta_2 = \delta_a = \{A \in I^X : A \leq \underline{a}\} \cup \{\underline{1}\}$. Then (X, δ_1, δ_2) is α-B -finitistic fuzzy bitopological space. For, clearly (X, δ_1, δ_2) is fuzzy bitopological space. Let $\mathcal{U}$ be any $\delta_i\alpha$ -open cover of (X, δ_1, δ_2). Then clearly, $\underline{1} \in \mathcal{U}$ (because no subfamily of δ_i can be a $\delta_i\alpha$ -open cover of (X, δ_1, δ_2) unless $\underline{1} \in \mathcal{U}$). Now clearly, $\mathcal{V} = \{\underline{0}, \underline{1}\}$ is a zero order δ_j α -open refinement of $\mathcal{U}$. Hence (X, δ_1, δ_2) is α-B -finitistic.

Theorem 2.9. *Every* α-B *-compact fuzzy bitopological space is* α-B *-finitistic.*

Proof. Let (X, δ_1, δ_2) be a α-B -compact fuzzy bitopological space. We have to show that it is α-B -finitistic. Let $\mathcal{U} = \{U_\lambda : \lambda \in \Lambda\}$ be any $\delta_i\alpha$ -open cover of X . Since (X, δ_1, δ_2) is α-B -compact, therefore $\mathcal{U}$ has a δ_j finite α -subcover say $\{U_1, U_2, U_3, \cdots, U_n\}$. Then $\mathcal{V} = \{\underline{0}, U_1, U_2, U_3, \cdots, U_n\}$ is clearly δ_j finite order α -open refinement of $\mathcal{U}$. Hence (X, δ_1, δ_2) is α-B -finitistic.

Definition 2.10. A fuzzy bitopolgoical space (X, δ_1, δ_2) is said to be α-B -paracompact if each $\delta_i\alpha$ -open cover of X has a δ_j locally finite α -open refinement.

Theorem 2.11. *Every finite dimensional* α-B *-paracompact fuzzy bitopological space is* α-B *-finitistic.*

Proof. Let (X, δ_1, δ_2) be a α-B -paracompact fuzzy bitopological space. We have to show that it is α-B -finitistic. Let $\mathcal{U} = \{U_\lambda : \lambda \in \Lambda\}$ be any $\delta_i\alpha$ -open cover of X . Since (X, δ_1, δ_2) is α-B -paracompact, therefore $\mathcal{U}$ has a δ_j locally finite α -subcover, say $\mathcal{V}$. Also since $\dim X < \infty$ and $\mathcal{V}$ is locally finite, it follows that $\mathcal{V}$ and hence $\mathcal{U}$ has a δ_j finite order α -open refinement. Hence (X, δ_1, δ_2) is α-B -finitistic.

Remark 2.12. Converse of above Theorem 2.9 is not true.
Consider the following example:

Example. Let X be an infinite set. Let $\delta_1 = \{\chi_U : U \subset X\}$ and $\delta_2 = \delta_1$. Then clearly, (X, δ_1, δ_2) is α-B -finitistic space. This is because $\mathcal{V} = \left\{\chi_{\{x\}} : x \in X\right\}$ is clearly δ_j finite order α -open refinement of every $\delta_i\alpha$ -open cover of X . But (X, δ_1, δ_2) is not α-B -compact because $\mathcal{V} = \left\{\chi_{\{x\}} : x \in X\right\}$ is a $\delta_i\alpha$ -open cover of (X, δ_1, δ_2) which has no δ_i finite α -subcover.

Theorem 2.13. *If (X,δ_1,δ_2) is a α-B -finitistic fuzzy bitopological space, then both (X,δ_1) and (X,δ_2) are α -finitistic.*

Proof. Suppose (X,δ_1,δ_2) is an α-B -finitistic fuzzy bitopological space. We have to show that both (X,δ_1) and (X,δ_2) are α -finitistic spaces. Let $\mathcal{U}=\{U_\lambda:\lambda\in\Lambda\}$ be any α -open cover of (X,δ_1). Then clearly $\mathcal{U}=\{U_\lambda:\lambda\in\Lambda\}$ is δ_1 α -open cover of (X,δ_1,δ_2). Since (X,δ_1,δ_2) is α-B -finitistic fuzzy bitopological space, therefore $\mathcal{U}$ has a δ_2 finite order α -open refinement, say $\mathcal{V}$. Again since $\mathcal{V}$ is $\delta_2\alpha$ -open cover of (X,δ_1,δ_2) and (X,δ_1,δ_2) is α-B -finitistic, therefore $\mathcal{V}$ has a δ_1 finite order α -open refinement, say $\mathcal{U}_1$. Then clearly $\mathcal{U}_1$ is a finite order α -open refinement of $\mathcal{U}$. Hence (X,δ_1) is α -finitistic. Similarly we can show that (X,δ_2) is α -finitistic.

Remark 2.14. Converse of above theorem is not true.
See the following example:

Example. Let $X=\{a,b\}$ be a set having two elements. Let $\delta_1=\{\underline{0},\underline{1}\}$ and $\delta_2=\left\{\underline{0},\chi_{\{a\}},\chi_{\{b\}},\underline{1}\right\}$. Then clearly both (X,δ_1) and (X,δ_2) are α -finitistic fuzzy topological spaces. But (X,δ_1,δ_2) is not α-B -finitistic because $\left\{\chi_{\{a\}},\chi_{\{b\}}\right\}$ is δ_2 α -open cover of (X,δ_1,δ_2) which has no δ_1 finite order α -open refinement.

Definition 2.15. A fuzzy bitopological space (X,δ_1,δ_2) is said to be α -finitistic if each $\delta_i\alpha$ -open cover of X has a δ_i finite order α -open refinement.

Theorem 2.16. *Let (X,δ_1,δ_2) be a fuzzy bitopological space, where X is a finite set. Then (X,δ_1,δ_2) is α -finitistic.*

Proof. Let (X,δ_1,δ_2) be any fuzzy bitopological space. Let $\mathcal{U}=\{U_\lambda:\lambda\in\Delta\}$ be a $\delta_i\alpha$ -open cover of (X,δ_1,δ_2). Since X is finite, we can write, $X=\{n_1,n_2,\cdots,n_k\}$. Since $\mathcal{U}$ is $\delta_i\alpha$ -open cover of (X,δ_1,δ_2), there exists some $U_{\lambda_i}\in\mathcal{U}$ such that $U_{\lambda_i}(n_i)>\alpha$, $\forall i=1,2,\cdots,k$. Then clearly, $\mathcal{V}=\left\{\underline{0},U_{\lambda_1},U_{\lambda_2},\cdots,U_{\lambda_n}\right\}$ is δ_i finite order α -open refinement of $\mathcal{U}$. Hence (X,δ_1,δ_2) is α -finitistic.

Remark 2.17. If X is finite, then (X,δ_1,δ_2) need not be α-B -finitistic.

Example. Let $X=\{a,b\}$. Let $\delta_1=\{\underline{0},\underline{1}\}$ and $\delta_2=\left\{\underline{0},\chi_{\{a\}},\chi_{\{b\}},\underline{1}\right\}$. Then (X,δ_1,δ_2) is not α-B -finitisitc. This is because $\left\{\chi_{\{a\}},\chi_{\{b\}}\right\}$ is $\delta_2\alpha$ -open cover of (X,δ_1,δ_2) which has no δ_1 finite order α -open refinement.

Remark 2.18. An α -finitistic fuzzy bitopological space need not be α-B -finitistic. Consider the following example:

Example. Let $X = \{a, b\}$ be a set having two elements. Let $\delta_1 = \{\underline{0}, \underline{1}\}$ and $\delta_2 = \{\underline{0}, \chi_{\{a\}}, \chi_{\{b\}}, \underline{1}\}$. Then clearly (X, δ_1, δ_2) is a fuzzy bitopological space. It is clear that every δ_1 (or δ_2) open cover of (X, δ_1, δ_2) has δ_1 (or δ_2) finite order open refinement. Hence (X, δ_1, δ_2) is α -finitistic. But (X, δ_1, δ_2) is not α-B -finitistic because $\{\chi_{\{a\}}, \chi_{\{b\}}\}$ is $\delta_2\alpha$ -open cover of (X, δ_1, δ_2) which has no δ_1 finite order α -open refinement.

Remark 2.19. An α-B -finitistic fuzzy bitopological space need not be α -finitistic.

Definition 2.20. A fuzzy bitopological space (X, δ_1, δ_2) is said to be B -finitistic if each δ_i open cover of X has a δ_j finite order open refinement.

Remark 2.21. An α-B -finitistic fuzzy bitopological space need not be B -finitistic. Consider the following example:

Example. Let X be an infinite set. Let $\delta_1 = \{\chi_U : U \subset X\}$ and $\delta_2 = \delta_1$. Then clearly, (X, δ_1, δ_2) is a fuzzy bitopological space. Since $\mathcal{V} = \{\chi_{\{x\}} : x \in X\}$ is δ_i finite order α -open refinement of every $\delta_i\alpha$ -open cover of X, it follows that (X, δ_1, δ_2) is α-B -finitistic space. But (X, δ_1, δ_2) is not B -finitistic because the δ_i open cover $\mathcal{V} = \{\chi_{\{x\}} : x \in X\}$ of (X, δ_1, δ_2) which has no δ_j finite order open refinement.

Theorem 2.22. *Let (X, δ_1, δ_2) be an α-B -finitistic fuzzy bitopological space where either of δ_1 or δ_2 is discrete fuzzy topology on X. Then $\delta_1 = \delta_2$.*

Proof. Proof is easy and hence is omitted.

Definition 2.23. Let (X, τ_1, τ_2) and (Y, τ_3, τ_4) be two bitopological spaces. A function $f : (X, \tau_1, \tau_2) \to (Y, \tau_3, \tau_4)$ is said to be α-B -continuous if inverse image of every $\tau_3\alpha$ -open subset of Y is $\tau_2\alpha$ -open subset of X and inverse image of every $\tau_4\alpha$ -open subset of Y is $\tau_1\alpha$ -open subset of X.

Definition 2.24. Let (X, τ_1, τ_2) an (Y, τ_3, τ_4) be two bitopological spaces. A function $f : (X, \tau_1, \tau_2) \to (Y, \tau_3, \tau_4)$ is said to be α-B -homeomorphism if both f and f^{-1} are α-B -continuous.

Remark 2.25. An α-B -continuous image of α-B -finitistic fuzzy bitopological space need not be α-B -finitistic.

Consider the following example:

Example. Let $X=\{a,b\}$ be a set having two elements. Let $\delta_1=\{\underline{0},\chi_{\{a\}},\chi_{\{b\}},\underline{1}\}$ and $\delta_2=\{\underline{0},\underline{1}\}$. Then clearly both δ_1 and δ_2 are fuzzy topologies on X. Then (X,δ_1,δ_2) and (X,δ_1,δ_2) are fuzzy bitopological spaces. Here (X,δ_1,δ_2) is α-B -finitistic but (X,δ_1,δ_2) is not α-B -finitistic because $\{\chi_{\{a\}},\chi_{\{b\}}\}$ is $\delta_1\alpha$ -open cover of X which has no δ_2 finite order α -open refinement. Let $I:X\to X$ be the identity function. Then $I:(X,\delta_1,\delta_2)\to(X,\delta_1,\delta_2)$ is α-B -continuous. It means (X,δ_1,δ_2) is α-B -continuous image of (X,δ_1,δ_2). Hence (X,δ_1,δ_2) is α-B -finitistic but (X,δ_1,δ_2) is not α-B -finitistic.

Remark 2.26. α-B -continuous inverse image of α-B -finitistic fuzzy bitopological space need not be α-B -finitistic.

See the following example.

Example. Let $X=\{a,b\}$, $\delta_1=\{\underline{0},\chi_{\{a\}},\underline{1}\}$ and $\delta_2=\{\underline{0},\chi_{\{a\}},\chi_{\{b\}},\underline{1}\}$. Then (X,δ_1,δ_2) is a fuzzy bitopological space. But it is not α-B -finitistic. Let $Y=\{x,y\}$, $\delta_3=\{\underline{0},\chi_{\{x\}},\underline{1}\}$ and $\delta_4=\{\underline{0},\chi_{\{y\}},\underline{1}\}$. Then (Y,δ_3,δ_4) is a fuzzy bitopological space and it is α-B -finitistic. Define $f:X\to Y$ as $f(a)=x$ and $f(b)=y$. Then clearly $f:(X,\delta_1,\delta_2)\to(Y,\delta_3,\delta_4)$ is α-B -continuous. Here (Y,δ_3,δ_4) is α-B -finitistic but (X,δ_1,δ_2) which is α-B -continuous inverse image of (Y,δ_3,δ_4) is not α-B -finitistic.

Theorem 2.27. *α-B -Homeomorphic image of α-B -finitistic fuzzy bitopological space is α-B -finitistic.*

Proof. Let $f:(X,\delta_1,\delta_2)\to(Y,\delta_3,\delta_4)$ be an α-B -homeomorphism. Suppose that (X,δ_1,δ_2) is α-B -finitistic fuzzy bitopological space. We have to show that (Y,δ_3,δ_4) is α-B -finitistic. Let $\mathcal{U}=\{U_\lambda:\lambda\in\Lambda\}$ be any $\delta_3\alpha$ -open cover of Y. Since $f:(X,\delta_1,\delta_2)\to(T,\delta_3,\delta_4)$ is α-B -continuous, therefore each $U_\lambda f$ is δ_2 α -open subset of X. We shall show that $\mathcal{V}=\{U_\lambda f:U_\lambda\in\mathcal{U}\}$ is $\delta_2\alpha$ -open cover of X. For this let $x\in X$. Then $f(x)\in Y$. Since $\mathcal{U}$ is $\delta_3\alpha$ -open cover of Y, there exists $U_\lambda\in\mathcal{U}$ such that $U_\lambda(f(x))>\alpha$. But $U_\lambda(f(x))>\alpha\Rightarrow(U_\lambda f)(x)>\alpha$. This shows that $\mathcal{V}$ is $\delta_2\alpha$ -open cover of X. Since (X,δ_1,δ_2) is α-B -finitistic fuzzy bitopological space, therefore $\mathcal{V}$ has δ_1 finite order α -open refinement say $\mathcal{V}_1$. We now claim that $\mathcal{U}_1=\{Wf^{-1}:W\in\mathcal{V}_1\}$ is a δ_4 finite order α -open refinement of $\mathcal{U}$.

Since $f^{-1}:(Y,\delta_3,\delta_4)\to(X,\delta_1,\delta_2)$ is α-B -continuous, therefore, Wf^{-1} is δ_4 fuzzy open subset of (Y,δ_3,δ_4). Also, let $y\in Y$. Then there exists $x\in X$ such that $y=f(x)$. Since f is bijective, therefore $y=f(x)$ implies $x=f^{-1}(y)$.

Since $x\in X$ and $\mathcal{V}_1$ is a $\delta_1\alpha$ -open cover of (X,δ_1,δ_2), there exists some $W\in\mathcal{V}_1$ such that $W(x)>\alpha$. But $W(x)>\alpha\Rightarrow W(f^{-1}(y))>\alpha\Rightarrow(Wf^{-1})(y)>\alpha$.

This implies that $\mathcal{U}_1$ is δ_4 α -open cover of (Y,δ_3,δ_4).

We now show that $\mathcal{U}_1$ refines $\mathcal{U}$.

Let $Wf^{-1}\in\mathcal{U}_1$. Then $W\in\mathcal{V}_1$. Since $\mathcal{V}_1$ refines $\mathcal{V}$, there exists some $U_\lambda\in\mathcal{V}$ such that $W\le U_\lambda$. But $W\le U_\lambda$ implies $Wf^{-1}\le U_\lambda f^{-1}$. This implies that $\mathcal{U}_1$ refines $\mathcal{U}$. Since $\mathcal{U}_1$ is of finite order, it is easy to check that order of $\mathcal{U}_1$ is also finite. This proves that $\mathcal{U}_1=\{Wf^{-1}:W\in\mathcal{V}_1\}$ is a δ_4 finite order α -open refinement of $\mathcal{U}$.

Similarly we can show that each δ_4 α -open cover of Y has a δ_3 finite order α -open refinement. Hence (Y,δ_3,δ_4) is α-B -finitistic.

Definition 2.28. A fuzzy topological space (X,δ) is said to be weakly induced if for all $U\in\delta$ and $a\in I$, $U_{(a)}\in[\delta]$.

Definition 2.29. A fuzzy bitopological space (X,δ_1,δ_2) is said to be weakly induced if both the fuzzy topological spaces (X,δ_1) and (X,δ_2) are weakly induced.

Theorem 2.30. *Let (X,δ_1,δ_2) be a weakly induced fuzzy bitopological space. Then (X,δ_1,δ_2) is α-B -finitistic if and only if $(X,[\delta_1],[\delta_2])$ is B -finitistic.*

Proof. Suppose (X,δ_1,δ_2) is a α-B -finitistic weakly induced fuzzy bitopological pace. We have to show that $(X,[\delta_1],[\delta_2])$ is B -finitistic. Let $\mathcal{U}=\{U_\lambda:\lambda\in\Lambda\}$ be any $[\delta_i]$ open cover of $(X,[\delta_1],[\delta_2])$. We show that $\mathcal{V}=\{\chi_{U_\lambda}:U_\lambda\in\mathcal{U}\}$ is a $\delta_i\alpha$ -open cover of (X,δ_1,δ_2). By defineition of $[\delta_i]$, each $\chi_{U_\lambda}\in\delta_i$, for each $U_\lambda\in[\delta_i]$. Let $x\in X$. Since $\mathcal{U}=\{U_\lambda:\lambda\in\Lambda\}$ is a $[\delta_i]$ open cover of $(X,[\delta_1],[\delta_2])$, there exists some $U_\lambda\in\mathcal{U}$ such that $x\in U_\lambda$. But $x\in U_\lambda\Rightarrow\chi_{U_\lambda}(x)=1>\alpha\Rightarrow\chi_{U_\lambda}(x)>\alpha$. This implies that $\mathcal{V}=\{\chi_{U_\lambda}:U_\lambda\in\mathcal{U}\}$ is a $\delta_i\alpha$ -open cover of (X,δ_1,δ_2). Since (X,δ_1,δ_2) is α-B -finitistic, therefore $\mathcal{V}$ has a δ_j finite order α -open refinement say $\mathcal{V}_1=\{W_t:t\in\Delta\}$. We show that $\mathcal{U}_1=\{(W_t)_{(0)}:W_t\in\mathcal{V}_1\}$ is a $[\delta_j]$ finite order open refinement of $\mathcal{U}$. Since (X,δ_1,δ_2) is weakly induced, therefore each $(W_t)_{(0)}\in[\delta_j]$. Let $x\in X$. Suppose $x\notin(W_t)_{(0)}$, for all $(W_t)_{(0)}\in\mathcal{U}_1$. Then $W_t(x)=0$ for all $W_t\in\mathcal{V}_1$.

But $W_t(x)=0$ for all $W_t \in \mathcal{V}_1 \Rightarrow \{W_t : t \in \Delta\}$ is not a $\delta_j\alpha$ -open cover (X,δ_1,δ_2). This is a contradiction. Hence $x \in (W_t)_{(0)}$ for some $(W_t)_{(0)} \in \mathcal{U}_1$. It means $\mathcal{U}_1 = \{(W_t)_{(0)} : W_t \in \mathcal{V}_1\}$ is a $[\delta_j]$ open cover of $(X,[\delta_1],[\delta_2])$. Let $(W_t)_{(0)} \in \mathcal{U}_1$. Let $x \in (W_t)_{(0)}$. Then $W_t(x)>0$. Since $\mathcal{V}_1$ is a refinement of $\mathcal{V}$, there exists some $\chi_{U_\lambda} \in \mathcal{V}$ such that $W_t < \chi_{U_\lambda}$. But $W_t(x)>0$ and $W_t < \chi_{U_\lambda} \Rightarrow \chi_{U_\lambda}(x)=1 \Rightarrow x \in U_\lambda \Rightarrow (W_t)_{(0)} \subset U_\lambda$. This implies that $\mathcal{U}_1$ is refinement of $\mathcal{U}$. Now we show that order of $\mathcal{U}_1$ is finite. Here order of $\mathcal{V}_1$ is finite. Let order of $\mathcal{V}_1 = n$. Let $\{(W_1)_{(0)},(W_2)_{(0)},(W_3)_{(0)},\cdots,(W_{n+2})_{(0)}\}$ be any subfamily of $\mathcal{U}_1$ having $n+2$ elements. We show that $\bigcap_{i=1}^{n+2}(W_i)_{(0)} \neq \emptyset$. Let $\bigcap_{i=1}^{n+2}(W_i)_{(0)} \neq \emptyset$. Then there exists some $x\in \bigcap_{i=1}^{n+2}(W_i)_{(0)}$. But $x\in \bigcap_{i=1}^{n+2}(W_i)_{(0)} \Rightarrow x \in (W_i)_{(0)}$ for all $i=1,2,3,\cdots,n+2 \Rightarrow W_i(x)>0$ for all $i=1,2,3,\cdots,n+2 \Rightarrow \bigwedge_{i=1}^{n+2}(W_i)(x) > 0 \Rightarrow \bigwedge_{i=1}^{n+2}(W_i)(x) \neq 0$. This implies that order of $\mathcal{V}_1$ is exceeding n. This is a contradiction. Hence order of $\mathcal{U}_1$ is not exceeding n. It means order of $\mathcal{U}_1$ is finite. This proves that $\mathcal{U}_1$ is a $[\delta_j]$ finite order open refinement of $\mathcal{U}$. Hence $(X,[\delta_1],[\delta_2])$ is B -finitistic.

Conversely, suppose $(X,[\delta_1],[\delta_2])$ is B -finitistic. We have to show that (X,δ_1,δ_2) is α-B -finitistic. Let $\mathcal{U} = \{U_\lambda : \lambda \in \Delta\}$ be a $\delta_i\alpha$ -open cover of (X,δ_1,δ_2). We shall show that $\mathcal{V} = \{(U_\lambda)_{(\alpha)} : U_\lambda \in \mathcal{U}\}$ is δ_i open cover of $(X,[\delta_1],[\delta_2])$. Since $\mathcal{U}$ is $\delta_i\alpha$ -open cover of (X,δ_1,δ_2), for every $x \in X$, there exists $U_\lambda \in \mathcal{U}$ such that $U_\lambda(x) > \alpha$. Now, $U_\lambda(x) > \alpha \Rightarrow x \in (U_\lambda)_{(\alpha)}$. Thus for every $x \in X$, there exists $(U_\lambda)_{(\alpha)} \in \mathcal{V}$ such that $x \in (U_\lambda)_{(\alpha)}$. Hence $\mathcal{V}$ is δ_i open cover of $(X,[\delta_1],[\delta_2])$. Since $(X,[\delta_1],[\delta_2])$ is B -finitistic, $\mathcal{V}$ has a δ_j finite order open refinement, say $\mathcal{V}_1 = \{V_\beta : \beta \in \Delta_1\}$. Let V'_λ be the union of all those members of $\mathcal{V}_1$ which are subsets of $(U_\lambda)_{(\alpha)}$. Then clearly, $\mathcal{V}_1' = \{V'_\lambda : \lambda \in \Delta_1\}$ is a δ_j open refinement of $\mathcal{V}$ and order of $\mathcal{V}_1'$ is finite. Let $\mathcal{U}_1 = \{\chi_{V'_\lambda} : V'_\lambda \in \mathcal{V}_1'\}$. We show that $\mathcal{U}_1$ is $\delta_j\alpha$ -open cover of (X,δ_1,δ_2). For this, let $x \in X$. Since $\mathcal{V}_1'$ is a δ_j open cover of $(X,[\delta_1],[\delta_2])$, there exists $V'_\lambda \in \mathcal{V}_1'$ such that $x \in V'_\lambda$. But $x \in V'_\lambda \Rightarrow x \in (U_\lambda)_{(\alpha)} \Rightarrow U_\lambda(x) > \alpha$. Also, $x \in V'_\lambda \Rightarrow \chi_{V'_\lambda} = 1$. Therefore, $(\chi_{V'_\lambda})(x) = \chi_{V'_\lambda}(x) \wedge U_\lambda(x) > 1 \wedge \alpha = \alpha$. This shows that $\mathcal{U}_1$ is $\delta_j\alpha$ -open cover of (X,δ_1,δ_2). Clearly, $\mathcal{U}_1$ is α -open refinement of $\mathcal{U}$. Since $\mathcal{V}_1'$ is of finite order, we can assume that order of $\mathcal{V}_1' = n$ (say). Then it is easy to show that order of $\mathcal{U}_1$ is not exceeding n. Hence (X,δ_1,δ_2) is α-B -finitistic.

Theorem 2.31. *Let (X,τ_1,τ_2) be a general bitopological space. Then (X,τ_1,τ_2) is B-finitistic if and only if $(X,\omega(\tau_1),\omega(\tau_2))$ is α-B-finitistic, where ω is the Lowen functor*.

Proof. We know that $(X,\omega(\tau_1),\omega(\tau_2))$ is weakly induced fuzzy bitopological space and $[\omega(\tau_i)]=\tau_i$, for $i=1,2$. By above Theorem 2.30, (X,τ_1,τ_2) is B-finitistic if and only if $(X,\omega(\tau_1),\omega(\tau_2))$ is B-finitistic.

Theorem 2.32. *A fuzzy bitopological space (X,δ_1,δ_2) is α-B-finitistic if and only if $(X,\iota_\alpha(\delta_1),\iota_\alpha(\delta_2))$ is B-finitistic.*

Proof. We know that $\iota_\alpha(\delta_i)=\{U_{(\alpha)}:U\in\delta_i\}$, for $i=1,2$.

Suppose (X,δ_1,δ_2) is α-B-finitistic. We shall show that $(X,\iota_\alpha(\delta_1),\iota_\alpha(\delta_2))$ is B-finitistic. For this, let $\mathcal{U}=\{U_\lambda:\lambda\in\Delta\}$ be any δ_i open cover of $(X,\iota_\alpha(\delta_1),\iota_\alpha(\delta_2))$. Then for each $U_\lambda\in\mathcal{U}$, there exists $V_\lambda\in\delta_i$ such that $U_\lambda=(V_\lambda)_{(\alpha)}$. We shall show that $\mathcal{V}=\{V_\lambda:U_\lambda=(V_\lambda)_\alpha\in\mathcal{U}\}$ is a $\delta_i\alpha$-open cover of (X,δ_1,δ_2). For this, let $x\in X$. Since $\mathcal{U}$ is δ_i open cover of $(X,\iota_\alpha(\delta_1),\iota_\alpha(\delta_2))$, there exists $U_\lambda\in\mathcal{U}$ such that $x\in U_\lambda$. Now, $x\in U_\lambda\Rightarrow x\in(V_\lambda)_{(\alpha)}\Rightarrow V_\lambda(x)>\alpha$. Thus for $x\in X$, there exists $V_\lambda\in\mathcal{V}$ such that $V_\lambda(x)>\alpha$. Hence $\mathcal{V}$ is $\delta_i\alpha$-open cover of X. Since (X,δ_1,δ_2) is α-B-finitistic, $\mathcal{V}$ has δ_j finite order α-open refinement, say $\mathcal{V}_1=\{W_\beta:\beta\in\Delta_1\}$. Then clearly, $\mathcal{U}_1=\{(W_\beta)_{(\alpha)}:W_\beta\in\mathcal{V}_1\}$ is δ_j finite order open refinement of $\mathcal{U}$. Hence $(X,\iota_\alpha(\delta_1),\iota_\alpha(\delta_2))$ is B-finitistic.

Conversely, suppose $(X,\iota_\alpha(\delta_1),\iota_\alpha(\delta_2))$ is B-finitistic. We have to show that (X,δ_1,δ_2) is α-B-finitistic. Let $\mathcal{U}=\{U_\lambda:\lambda\in\Delta\}$ be any $\delta_i\alpha$-open cover of (X,δ_1,δ_2). It is easy to show that $\mathcal{V}=\{(U_\lambda)_{(\alpha)}:U_\lambda\in\mathcal{U}\}$ is a δ_i open cover of $(X,\iota_\alpha(\delta_1),\iota_\alpha(\delta_2))$. Since $(X,\iota_\alpha(\delta_1),\iota_\alpha(\delta_2))$ is B-finitistic, $\mathcal{V}$ has a δ_j finite order open refinement, say $\mathcal{V}_1=\{(W_\beta)_{(\alpha)}:\beta\in\Delta_1\}$. Then clearly, $\mathcal{U}_1=\{W_\beta:(W_\beta)_{(\alpha)}\in\mathcal{V}_1\}$ is a δ_j finite order α-open refinement of $\mathcal{U}$. Hence (X,δ_1,δ_2) is α-B-finitistic.

Theorem 2.33. *Let (X,δ_1,μ_1) and (Y,δ_2,μ_2) be two α-B-finitistic bitopological spaces. Then $(X\cup Y,\delta_1\oplus\delta_2,\mu_1\oplus\mu_2)$ is α-B-finitistic.*

Proof. Suppose (X,δ_1,μ_1) and (Y,δ_2,μ_2) are two α-B -finitistic bitopological spaces. We have tow show that $(X\cup Y,\delta_1\oplus\delta_2,\mu_1\oplus\mu_2)$ is α-B -finitistic. For this, let $\mathcal{U}=\{U_\lambda:\lambda\in\Delta\}$ be a $\delta_1\oplus\delta_2$ α -open cover of $(X\cup Y,\delta_1\oplus\delta_2,\mu_1\oplus\mu_2)$. Then clearly, $\mathcal{U}|_X=\{U_\lambda|_X:U_\lambda\in\mathcal{U}\}$ and $\mathcal{U}|_Y=\{U_\lambda|_Y:U_\lambda\in\mathcal{U}\}$ are δ_1 and δ_2 α -open covers of (X,δ_1,μ_1) and (Y,δ_2,μ_2), respectively. Since both (X,δ_1,μ_1) and (Y,δ_2,μ_2) are α-B -finiitistic, therefore, $\mathcal{U}|_X$ and $\mathcal{U}|_Y$ have μ_1 and μ_2 finite order α -open refinements, say $\mathcal{V}_X$ and $\mathcal{V}_Y$, respectively. Define $R_b=V_b$ on X & $R_{b'}=\underline{0}$ on Y and $S_t=W_t$ on Y & $S_{t'}=\underline{0}$ on X, where $V_b\in\mathcal{V}_X$ and $W_t\in\mathcal{V}_Y$.

Then clearly, the family of all R_b's and S_t's defined above is a $\mu_1\oplus\mu_2$ finite order α -open refinement of $\mathcal{U}$. Hence $(X\cup Y,\delta_1\oplus\delta_2,\mu_1\oplus\mu_2)$ is α-B -finitiistic.

Theorem 2.34. *Let $\{(X_t,\delta_t,\mu_t):t\in T\}$ be a family of fuzzy bitopological spaces such that $(X,\oplus_{t\in T}\delta_t,\oplus_{t\in T}\mu_t)$ is α-B -finitistic, where $X=\bigcup_{t\in T}X_t$. Then (X_t,δ_t,μ_t) is α-B -finitisitc, $\forall t\in T$.*

Proof. Here, $X=\bigcup_{t\in T}X_t$, where X_t's are disjoint. Suppose $(X,\oplus_{t\in T}\delta_t,\oplus_{t\in T}\mu_t)$ is α-B -finitistic. Let $\mathcal{U}_t=\{U_\lambda:\lambda\in\Delta\}$ be any $\oplus_{t\in T}\delta_t\alpha$ -open cover of (X_t,δ_t).

For all, $U_\lambda\in\mathcal{U}_t$, we define $R_\lambda=U_\lambda$ on X_t and $R_\lambda=\underline{1}$ on $X-X_t$. Then clearly $\mathcal{U}$, the family of all R_λ's is $\delta_t\alpha$ -open cover of $(X,\oplus_{t\in T}\delta_t,\oplus_{t\in T}\mu_t)$. Since $(X,\oplus_{t\in T}\delta_t,\oplus_{t\in T}\mu_t)$ is α-B -finitistic, therefore, $\mathcal{U}$ has a $\oplus_{t\in T}\mu_t$ finite order α -open refinement, say $\mathcal{V}=\{V_\beta:\beta\in\Delta_1\}$. Then clearly, $\mathcal{V}_t=\{V_\beta|_{X_t}:V_\beta\in\mathcal{V}\}$ is μ_t finite order α -open refinement of $\mathcal{U}_t$. Hence (X_t,δ_t,μ_t) is α-B -finitistic, $\forall t\in T$.

Theorem 2.35. *The sum space $(X\cup Y,\delta_1\oplus\delta_2,\mu_1\oplus\mu_2)$ is α-B -finitistic if and only if (X,δ_1,μ_1) and (Y,δ_2,μ_2) are α-B -finitistic.*

Proof. It follows from Theorem 2.33 and 2.34.

Theorem 2.36. *A fuzzy bitopological space (X,δ_1,δ_2) is α-B -finitistic if and only if each δ_i basic α -open cover of (X,δ_1,δ_2) has a δ_j finite order α -open refinement.*

Proof. First suppose that each δ_i basic α -open cover of (X,δ_1,δ_2) has a δ_j finite order α -open refinement. We shall show that (X,δ_1,δ_2) is α-B -finitistic. For this, let $\mathcal{U}$ be any $\delta_i\alpha$ -open cover of (X,δ_1,δ_2). For each $U_\lambda\in\mathcal{U}$, let $\mathcal{A}_t$ be the family of all the basic α -open subsets of (X,δ_1,δ_2) whose join is U_λ. Let $\mathcal{V}$ be the union of all these families $\mathcal{A}_\lambda$'s. then clearly, $\mathcal{V}$ is a basic α -open cover of (X,δ_1,δ_2). By the given condition, $\mathcal{V}$ has a δ_j finite order α-open refinement, say $\mathcal{V}_1$. Then clearly, $\mathcal{V}_1$ is a δ_i finite order α-open refinement of $\mathcal{U}$. Hence (X, δ_1, δ_2) is α-B-finitistic.

Converse is trivial.

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