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COMPOSITE CONVOLUTION VOLTERRA OPERATORS: ANALYSIS AND APPLICATIONS

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Abstract. In this paper we consider the Composite Convolution Volterra operators on the space $L^2[0,1]$. We calculate the numerical range of composite convolution Volterra operators. The conditions for composite convolution Volterra operator to be contraction have been explored. In this paper an attempt has also been made to investigate applications of composite convolution Volterra operators in solving Volterra convolution type equations.

1. Introduction. Let (X, Ω, μ) be a σ-finite measure space. For each $f \in L^p(\mu), 1 \leq p < \infty$, there exists a unique $\phi^{-1}(\Omega)$ measurable function $E(f)$ such that $\int gf\,d\mu = \int gE(f)\,d\mu$ for every $\phi^{-1}(\Omega)$ measurable function g for which left integral exists. The function $E(f)$ is called conditional expectation of f with respect to the sub- algebra $\phi^{-1}(\Omega)$. For more details about expectation operator, one can refer to Parthasarathy (1977). Let $\phi : X \to X$ be a non-singular measurable transformation (i.e., $\mu(E) = 0 \Rightarrow \mu\phi^{-1}(E) = 0$). Then a composition transformation , for $1 \leq p < \infty$, $C_\phi : L^p(\mu) \to L^p(\mu)$ is defined by $C_\phi f = f o\phi$ for every $f \in L^p(\mu)$. In case C_ϕ is continuous, we call it a composition operator induced by ϕ. It is easy to see that C_ϕ is a bounded operator if and only if $f_0 = \frac{d\mu\phi^{-1}}{d\mu}$, the Radon-Nikodym derivative of the measure $\mu\phi^{-1}$ with respect to the measure μ, is essentially bounded. For more detail about composition operator we refer to Singh and Manhas (1993). Given $f, g \in L^1(\mathbb{R})$, then convolution of f and g, $f * g$ is defined by

$$f * g(x) = \int g(x-y) f(y) d(y).$$

where g is fixed, $k(x,y) = g(x-y)$ is a convolution kernel, and the integral operator defined by

$$I_k f(x) = \int k(x-y) f(y) d\mu(y)$$