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Composition Operators on Weighted Orlicz Sequence Spaces

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Abstract. In this paper, we study the boundedness of composition operators between any two weighted Orlicz sequence spaces

Keywords. Composition operators, Boundedness, weighted Orlicz sequence spaces.

1 Introduction

Let $\phi:[0,\infty)\to [0,\infty)$ be a Young function, that is, a nondecreasing continuous convex function for which $\phi(0)=0$ and $\lim_{x\to\infty}\phi(x)=\infty$. Let (X,Σ,μ) be a σ -finite and purely atomic measure space, that is

$$X = \bigcup_{n=1}^{\infty} A_n$$

where A_n are atoms with the measure $\mu(A_n) = a_n > 0$ for all $n \in \mathbb{N}$ and $w = \{w_n\}$ be a weight in X i.e positive and summable real valued sequence. Then the weighted Orlicz sequence space $l_n^o(\{a_n\})$ is defined as the space of all real sequences $f = \{f_n\}_{n=1}^{\infty}$ such that $I_{\phi,w}(\lambda f, \{a_n\}) < \infty$ for some $\lambda > 0$, where

$$I_{\phi,w}(f, \{a_n\}) = \sum_{n=1}^{\infty} \phi(|f_n|)w_n a_n.$$

This space is a Banach space with the norm

$$||f||_{\phi,w,\{a_n\}} = \inf{\{\lambda > 0 \mid I_{\phi,w}(f/\lambda, \{a_n\}) \le 1\}}.$$

Throughout the paper, we assume (X, Σ, μ) to be a σ -finite and purely atomic measure space with atoms $\{A_n\}$ of measure $\mu(A_n) = a_n > 0$ for any $n \in \mathbb{N}, \tau : X \to X$ to be a measurable non-singular transformation such that $\tau(X) = X$ and $b_n := \mu(\tau^{-1}(A_n))/\mu(A_n)$.

Composition operators on Orlicz spaces have also been studied in [3], [4], [5], [9] and [17]. The techniques used in this paper essentially depend on the conditions of embedding of one Orlicz space into another (see, [13, Page 48] and [19] for details).