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A VALUE DISTRIBUTION RESULT RELATED TO HAYMAN'S ALTERNATIVE

KULDEEP SINGH CHARAK AND ANIL SINGH

ABSTRACT. Motivated by Bloch's Principle, we prove a value distribution result for meromorphic functions which is related to Hayman's alternative in certain sense.

1. Introduction and Main Result

The reader is assumed to be familiar with the standard notations of Nevanlinna value distribution theory of meromorphic functions (one may refer to [4, 6]) such as $T(r, f)$, $m(r, f)$, $N(r, f)$, etc. We shall denote the class of all meromorphic functions on a domain D in $\mathbb{C}$ by $\mathcal{M}(D)$ and we shall write, '$\langle f, D\rangle \in \mathcal{P}$' for '$f \in \mathcal{M}(D)$ satisfies the property $\mathcal{P}$ on D'.

We say that $\phi \in \mathcal{M}(\mathbb{C})$ is a small function of $f \in \mathcal{M}(\mathbb{C})$ if $T(r, \phi) = S(r, f)$ as $r \to \infty$ possibly outside a set of r of finite linear measure.

W.K. Hayman proved the following 'Picard type' theorem, also known as Hayman's alternative:

Theorem 1.1. [7] *Let $f \in \mathcal{M}(\mathbb{C})$ and let $l \geq 1$. Suppose that $f(z) \neq 0$, and $f^{(l)}(z) - 1 \neq 0$ for all $z \in \mathbb{C}$. Then f is constant.*

A subfamily $\mathcal{F}$ of $\mathcal{M}(D)$ is said to be normal in D if every sequence of members of $\mathcal{F}$ contains a subsequence that converges locally uniformly (w.r.t. the spherical metric) in D. Recall Bloch's Principle (see[10, 11]): *A subfamily $\mathcal{F}$ of $\mathcal{M}(D)$ with $\langle f, D\rangle \in \mathcal{P}$ for each $f \in \mathcal{F}$ is likely to be normal on D if $\mathcal{P}$ reduces every $f \in \mathcal{M}(\mathbb{C})$ to a constant.* Neither Bloch's Principle nor its converse is true (see [1, 2, 3, 8, 10]).

According to Bloch's Principle, to every 'Picard type' theorem there corresponds a normality criterion. A normality criterion corresponding to Theorem 1.1 was proved by Y.Gu as follows:

Theorem 1.2. [5] *Let $\mathcal{F} \subseteq \mathcal{M}(D)$ and let $l \geq 1$. Suppose that $f(z) \neq 0$, and $f^{(l)}(z) - 1 \neq 0$ for all $z \in D$ and $f \in \mathcal{F}$. Then $\mathcal{F}$ is normal in D.*

The constants 0 and 1 in Theorem 1.1 and Theorem 1.2 can be replaced by arbitrary constants a and $b \neq 0$:

Theorem 1.3. [7] *Let $f \in \mathcal{M}(D)$ and let $l \geq 1$. Suppose that $f(z) \neq a$, and $f^{(l)}(z) - b \neq 0$ for all $z \in \mathbb{C}$, where $a, b \in \mathbb{C}$, $b \neq 0$. Then f is constant.*