Properties of Weighted Composition Operators on Cesaro Function Spaces

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Abstract

Weighted composition operator is an operator obtained by taking the product of two operators namely, composition operator and multiplication operator. In this paper we consider the weighted composition operators on Cesaro function spaces. Our main concern in this paper is to discuss certain properties of weighted composition operators on Cesaro function spaces. The norm of weighted composition operator is computed. The necessary and sufficient condition for weighted composition operator to commute with multiplication operator is explored. The conditions for weighted operators to be contraction and isometry are characterized.

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1. Introduction

For $1 \leq p < \infty$, the Cesaro function space $Ces_p[0,\infty)$ was considered by Shiue [15], Hassard and Hussein [11] and Sy, Zhang and Lee [18]. The space $Ces_\infty[0,1]$ appeared in 1948 and it is known as the Korenblyum Krein and Levin space K which can be seen in Cui and Hudzik [7]. Recently, Astashkin ([2], [3], [4]) proved that in contract to Cesaro sequence spaces, the Cesaro function spaces $Ces_p(X)$ on both $X = [0,1]$ and $X = [0,\infty)$ for $1 < p < \infty$ are not reflexive and they do not have the fixed point property. The Cesaro function space $Ces_p[0,1]$ is the set of all Lebesgue measurable real functions f on [0,1] such that

$$\|f\|_{Ces_p} = [\int_0^1 (\frac{1}{x}\int_0^x f(y)\,d\mu(y))^p\,d\mu(x)]^{1/p} < \infty.$$

Under the above norm $Ces_p[0,1]$ is a Banach space. Given two functions $f : X \to R$ and $\varphi : X \to X$, we can produce new function by composing them under certain conditions. The resulting function is denoted by $f \circ \varphi$ and the operator $f \to f \circ \varphi$, is a linear operator C_φ known as the composition operator induced by a mapping φ. Another way to produce new function